The dynamics of a high-pressure ac gas discharge between dielectric coated electrodes near breakdown threshold

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An analytical theory of a high-pressure gas discharge between two metal electrodes covered with an insulator is presented. The theory is applicable when the voltage applied to the gap between electrodes only slightly exceeds the breakdown voltage. Analytic expressions for the electric field, electron and ion current densities, as a function of time and space, and an analysis of the stability of the discharge are given. A detailed discussion of the role of metastables in the discharge dynamics is included. The discharge in a plasma display cell is used as an example to demonstrate the utility of the theory. © *1995 American Institute of Physics.*

I. INTRODUCTION

The dynamics of a high-pressure (>100 Torr) ac discharge was investigated earlier both experimentally and numerically in a number of papers.[1–5] They included numerical analysis of an ac discharge in pure neon[3] and in a Ne+0.1% Ar mixture.[4] A rather good understanding of the mechanism of such a discharge has been achieved. It was identified as the Townsend discharge with external parameters influenced by the discharge and varying during the discharge. Numerical analysis showed good agreement with experiment. However, there is still no analytical theory which combines various parameters of a system in a few scaling laws. Such a theory should be able to predict characteristics of the discharge such as the space distributions and time dependence of the electric field and electron current during the discharge, excitation rates, luminosity in visible and UV spectra as a function of time, gas composition, and pressure, geometry, applied voltage, etc.

The general analysis of this problem is very difficult even in a 1D geometry, especially for mixtures. As it is an inherently transient discharge, the electric field rapidly changes in time and space. The dynamics of the discharge depends on the gas composition in the mixture and population of excited species which at high pressures include not only metastables but also resonantly excited atoms and excited molecules. Ionization and excitation rates for every species depend on the whole gas composition and on the space coordinate near electrodes since the electron distribution function in these regions is strongly affected by the vicinity of the boundaries. Obviously, the problem in many aspects requires only kinetic consideration.

However, despite all the aforementioned difficulties, one can develop a rather advanced and useful analytical theory if one considers only situations when the applied voltage only slightly exceeds the breakdown voltage. Indeed, in this case the current never reaches high values, and the distortions of the electric field due to space charges are small, so one can neglect them (later we will discuss this subject in more detail). The magnitude of the electric field changes slowly in time and its gradient is small. If one also assumes that the distance between the electrodes is significantly larger than electron mean free path and the distance at which electrons gain energy comparable to the ionization energy, then one can neglect the spatial dependence in the electron distribution function and ionization and excitation rates related to the influence of the boundaries. In this case the kinetic consideration of electrons would be necessary only for calculating electron driven rates and the effective secondary emission coefficient.

In the current paper we limit our consideration to an ac discharge in a single-component noble gas. We develop a relatively simple analytical theory which provides insightful understanding of the dynamics of the ac discharge and the role of different parameters. We show that the dynamics of the discharge depends on the difference between the applied and breakdown voltages rather than on the applied voltage itself and obtain simple analytical formulas describing the dynamics in terms of the first and second Townsend coefficients and geometric parameters. In the forthcoming publication we will extend our theory to derive analytical expressions for the excitation and ionization rates in a two-component gas mixture valid in a wide range of partial pressures of different components and consider the dynamic of the discharge and the light output from the cell.

Without restricting ourselves to any specific device, for all estimations we shall use the values of parameters (gas pressure, applied voltage, or the distance between electrodes) typical to current plasma display panels (pdp) as they are the most promising application of an ac discharge. That is, we consider gas pressure about ~0.5 atm and a gap size about $\sim 10^{-2}$ cm; thickness of the dielectric is usually about $\sim 10^{-3}$ cm.

Let us consider two parallel electrodes covered with an insulating material separated by a small gap. The gap is filled with a gas at a relatively high pressure. We shall refer to such a device as a "pdp" or "ac" cell. Some low level of ionization of the gas is provided initially by an external source. The distance L between electrodes is much larger than electron mean free path in the gas (λ_e). When an electric field is applied to the electrodes the electrons and ions move toward appropriate electrodes. Ions reaching the cathode may produce secondary electrons when they strike the surface of the cathode. If the electric field is high enough, these electrons

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on their way toward the anode may produce additional electrons and ions by ionization cascades. If the number of ions produced in the cascade, initiated by any one of secondary electrons from the surface, exceeds the number of ions necessary to produce that one electron due to a secondary emission, then the current grows *in time* and a breakdown occurs. Since the electrodes are covered with an insulator, a charge builds up on them and affects the electric field in the gap. As the field drops the ionization cascade becomes weak, and if the number of secondaries becomes less than in the previous pulse, the current decreases and the discharge self-extinguishes. Note that in our case the discharge extinguishes itself automatically, and does not require a change in the applied voltage as in an rf discharge or in a dc-discharge devices.

The property of a cell to accumulate and maintain a charge makes it possible to use it as both a light source and a memory element and is used in plasma displays. If, after the discharge has self-extinguished, one applies a voltage opposite in polarity to the previous one, the discharge may start again even if the applied voltage is less than the breakdown voltage. Obviously, the condition for starting a new discharge is that the applied voltage plus the voltage due to the stored charge exceeds the breakdown voltage.

In this paper, as we already mentioned, we consider a single discharge pulse in a monatomic gas when the applied voltage only slightly exceeds the breakdown voltage. To be more specific, let us recall the Townsend breakdown condition[6]

$$\Delta \equiv \gamma(e^{\alpha L}-1)-1=0, \tag{1}$$

where α and γ are the first and the second Townsend coefficients and L is the distance between electrodes. The discharge current grows when Δ is positive and decreases when Δ is negative. In this paper we assume that Δ is initially positive and much less than unity:

$$0<\gamma(e^{\alpha L}-1)-1 \ll 1. \tag{2}$$

Using a hydrodynamic approximation we find an analytic expression for the $I-V$ curve and the temporal and space characteristics of a discharge, and investigate its stability when a periodic potential is applied to the electrodes.

In Sec. II we give the set of equations which describe the pdp cell. In Sec. III we solve these equations and find the spatial distribution of electron and ion currents as functions of external parameters. Using these solutions in Sec. IV we obtain the $I-V$ characteristic curve and the temporal dependence of the current (the pulse shape). In Sec. V we investigate the stability of the discharge when an alternating square-wave voltage is applied to a cell. We then show that near the breakdown threshold the high-pressure discharge is unstable. In Sec. VI we give a qualitative discussion of the role of metastables and in Sec. VII we summarize the results obtained.

II. BASIC EQUATIONS

Consider a gas discharge cell consisting of two parallel flat electrodes covered with a thin layer of dielectric material. The z axis is directed from the anode toward the cathode, as

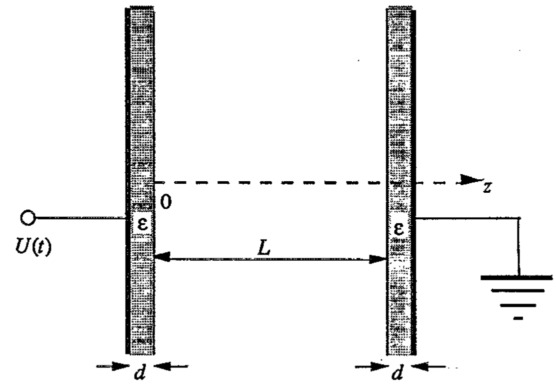

FIG. 1. A schematic diagram of a simple pdp cell.

shown in Fig. 1. We denote the dielectric constant of the film as ϵ, considering it as constant within the layer and assume that the thickness of the coating d is much less than the distance between electrodes $L(d \ll L)$.

Let $j_e(z,t)$ and $j_i(z,t)$ be the electron and ion current densities and $n_{e,i}$ and $v_{e,i}$ be their number densities and drift velocities. Drift velocities and the electric field have only one component—parallel to the z axis. According to our choice of the direction of the z axis, we have $E_z=E$, $v_i \equiv v_{iz}=|v_i|>0$, $v_e \equiv v_{ez}=-|v_e|<0$. The set of equations which describes our problem is[7]

$$\frac{\partial n_e}{\partial t}+\frac{\partial n_e v_e}{\partial z}=-\alpha(E)n_e v_e, \tag{3}$$

$$\frac{\partial n_i}{\partial t}+\frac{\partial n_i v_i}{\partial z}=-\alpha(E)n_e v_e, \tag{4}$$

$$div\,\epsilon\mathbf{E} \equiv \frac{\partial}{\partial z}\epsilon(z)E=4\pi e(n_i-n_e), \tag{5}$$

$$j_{i,e}=\pm e n_{i,e} v_{i,e}, \tag{6}$$

$$v_{i,e}=\pm\mu_{i,e}(E)E_z-D_{i,e}\frac{\nabla n_{i,e}}{n_{i,e}}$$

$$=\pm\mu_{i,e}(E)E-\frac{D_{i,e}}{n_{i,e}}\frac{\partial}{\partial z}n_{i,e}, \tag{7}$$

where $\alpha(E)$ is the first Townsend coefficient, e the elementary charge, $\mu_{i,e}(E)$ the ion and electron mobilities, and $D_{i,e}$ the ion and electron diffusion coefficients. The signs in the Eq. (6) relate to ions (upper sign) and electrons (lower sign), respectively. Equations (3) and (4) are the continuity equations with the source term describing ionization due to electron impact. Equation (5) is the Poisson equation for the electric field. Equation (7) assumes that electrons and ions reach their equilibrium drift velocities on time and space scales which are much shorter than those characterizing the changing electric field, so that $v_{e,i}$ are the functions of the local parameters—electric field and density gradients. The specific constraints resulted from these assumptions are derived in Appendix A.

Since the diffusion coefficient is proportional to an average energy of a particle, it is very small for ions because their energy can not differ significantly from a gas temperature in a high-density gas. Thus we neglect the diffusion of ions ($D_i=0$) and have

$$v_i=\mu_i(E)E, \tag{8}$$

so the ion current is determined only by the electric field.

The boundary conditions are (see Fig. 1)

$$j_e(L,t)=\gamma j_i(L,t), \quad j_i(0,t)=0, \quad U(t)=U_0\theta(t), \tag{9}$$

where $\theta(t)$ is a step function [$\theta(t)=0$ if $t<0$ and $\theta(t)=1$ if $t\geqslant 0$] and U_0 is the magnitude of the applied voltage.

We choose the following initial conditions:

$$n_{e,i}(z,0)=n_0, \quad \sigma_a(t=0)=-\sigma_c(t=0)=\sigma_0, \tag{10}$$

where $\sigma_c(t)\equiv\sigma(L,t)$ and $\sigma_a(t)\equiv\sigma(0,t)$ are the charges on the cathode and anode, respectively. Note that we consider a situation when all changes of the electric field during a discharge happened due to the discharge rather than changes of the external potential. The latter is correct if the time length of the discharge pulse is shorter than the half-period of the applied voltage, which in a pdp is typically about 5–10 μs. The result will justify this assumption.

III. THE DISTRIBUTIONS OF ELECTRON AND ION CURRENTS

We consider evolution times much longer than the electron drift time ($\tau\gg\tau_e\sim L/v_e$). Thus we can neglect the term containing the time derivative in Eq. (3) and one may rewrite Eq. (3) in the form

$$\frac{\partial j_e(z,t)}{\partial z}=-\alpha(E)j_e(z,t), \tag{11}$$

which has the solution

$$j_e(z,t)=j_e(L,t)\exp\left(\int_z^L \alpha(E)dz\right). \tag{12}$$

As was mentioned earlier, we assume that the voltage applied across the gap only slightly exceeds the breakdown voltage. In this case, according to Eqs. (5) and (6), the charges do not disturb the initial electric field in the gap and the solution of the Poisson equation (5) for the electric field in the gap with the boundary conditions (9) is straightforward. The electric field E is uniform in the gap between the electrodes and depends on time due to changes in the surface charge:

$$E(t)=\frac{U_0}{L+2d/\epsilon}+\frac{4\pi d}{\epsilon L+2d}[\sigma_a(t)-\sigma_c(t)]. \tag{13}$$

The charge densities on the electrodes can be expressed through the current densities near the electrodes:

$$\sigma_c(t)+\sigma_0=\int_0^t j(L,t')dt'=(1+\gamma)\int_0^t j_i(L,t')dt', \tag{14}$$

$$\sigma_a(t)-\sigma_0=-\int_0^t j(0,t')dt'=-\int_0^t j_e(0,t')dt'. \tag{15}$$

Using the solution for the electric field [Eq. (13)] we can perform the integration in Eq. (12) to find the electron current distribution $j_e(z,t)$:

$$j_e(z,t)=j_e(L,t)e^{\alpha(E)(L-z)}. \tag{16}$$

This solution shows that near the threshold the electron current has an exponential spatial dependence and always adjusts itself to its value on the cathode $j_e(L,t)$. However, to obtain $j_e(L,t)$ we need to know the ion current on the cathode [see the first boundary condition in Eq. (9)].

Let us now turn to the equation for the ion current. In this case the time derivative cannot be neglected since the drift time of the ions is significantly longer than that of electrons. We rewrite Eq. (4) in terms of the ion current rather than the ion density. Substituting $n_i=j_i/(ev_i)$ into Eq. (4) we obtain

$$\frac{1}{v_i}\frac{\partial j_i}{\partial t}+\frac{\partial j_i}{\partial z}-\frac{1}{v_i^2}\frac{\partial v_i}{\partial t}j_i=\alpha(E)j_e. \tag{17}$$

Using Eq. (8) we can estimate terms on the left-hand side of Eq. (17) as $j_i/(v_i\tau)$, j_i/L, and $(\Delta E/E)j_i/(v_i\tau)$, respectively, where ΔE is the net change of the electric field throughout the discharge. One can estimate the ratio $\Delta E/E$ as

$$\frac{\Delta E}{E}\sim\frac{U_0-U_{\rm br}}{U_{\rm br}}, \tag{18}$$

where $U_{\rm br}$ is the breakdown voltage. Since the applied voltage is close to the breakdown voltage, this parameter is small, and hence the term proportional to $\dot{E}\equiv dE/dt$ is small by a factor of $\dot{E}d/(Ev_i)$ and we neglect it. The solution of Eq. (17) can be easily obtained by integration along the characteristics

$$j_i(z,t)=\int_0^z \alpha[t-\tau_i(z,z')]j_e[z',t-\tau_i(z,z')]dz', \tag{19}$$

where $\alpha(t)\equiv\alpha[E(t)]$, and $\tau_i(z,z')=(z-z')/v_i$ is the ion drift time between a point z', where it was created to the point z. Note that Eq. (17) can be easily solved without neglecting the third term in the left-hand side of it. One can find this solution in Appendix B.

Using the solutions Eqs. (16) and (19) and the boundary condition $j_e(L,t)=\gamma j_i(L,t)$, we obtain an integral equation for determining $j_e(L,t)$:

$$j_e(L,t)=\gamma\int_0^L \alpha(t-z/v_i)j_e(L,t-z/v_i)e^{\alpha(t-z/v_i)z}\,dz. \tag{20}$$

By expanding $j_e(L,t-z/v_i)$ in a series,

$$j_e(L,t-z/v_i)=j_e(L,t)-\frac{\partial j_e(L,t)}{\partial t}z/v_i, \tag{21}$$

we finally obtain a differential equation for $j_e(L,t)$:

$$\frac{\partial j_e(L,t)}{\partial t}=j_e(L,t)v_i\frac{\gamma\int_0^L\alpha(t-z/v_i)e^{\alpha(t-z/v_i)z}\,dz-1}{\gamma\int_0^L\alpha(t-z/v_i)e^{\alpha(t-z/v_i)z}z\,dz}. \tag{22}$$

When performing the integrals in Eq. (22) we can neglect all the terms containing $\dot{E}$, (like $\dot{\alpha}\tau_i L$) as we did before when solving the Eq. (17) (later we will discuss the validity of this assumption). After this, the integrations in Eq. (22) become simple and we obtain

$$\frac{\partial j_e(L,t)}{\partial t}=j_e(L,t)\alpha(E)v_i\frac{\gamma[e^{\alpha(E)L}-1]-1}{\gamma[\alpha(E)L-1]e^{\alpha(E)L}+\gamma}. \tag{23}$$

To complete the system of differential equations for the current and electric field, let us differentiate Eq. (13) for the electric field. Using Eqs. (13)–(16) and the boundary condition (9), we obtain

$$\frac{\partial E}{\partial t}=-\frac{4\pi d}{\epsilon L+2d}[1+1/\gamma+e^{\alpha(E)L}]j_e(L,t). \tag{24}$$

Equation (23) together with Eq. (24) and initial conditions (10) complete the system for determining the time dependence of the current and electric field.

We now show that the characteristic rise time of the current τ is indeed much larger than the ion transit time $\tau_i=L/v_i$. Using Eq. (23) we can estimate the characteristic current rise time by assuming that the electric field during this time is constant:

$$\tau(E)=\frac{j_e(L,t)}{\partial j_e(L,t)/\partial t}=\frac{\gamma}{\alpha v_i}\frac{(\alpha L-1)e^{\alpha L}+1}{\gamma(e^{\alpha L}-1)-1}. \tag{25}$$

Using Eq. (2) in the form $e^{\alpha L}\approx 1+1/\gamma$ to rewrite the last term in the numerator, we obtain

$$\tau\sim\frac{1}{\alpha v_i}\frac{\gamma\alpha L e^{\alpha L}-1}{\gamma(e^{\alpha L}-1)-1}. \tag{26}$$

There are two limiting cases, small and large γ.

A. Small $\gamma\ll 1$

In this case $e^{\alpha L}\approx 1/\gamma\gg 1$ and

$$\tau\sim\frac{\alpha L-1}{\alpha v_i}[\gamma(e^{\alpha L}-1)-1]^{-1}\sim\frac{L}{v_i}[\gamma(e^{\alpha L}-1)-1]^{-1}$$

$$\gg\tau_i.$$

B. Large $\gamma\gg 1$

In this case $\alpha L\approx\gamma^{-1}\ll 1$ and

$$\tau\sim\frac{L}{2v_i}(\gamma\alpha L-1)^{-1}\gg\tau_i.$$

Thus in each case $\tau\gg\tau_i$.

IV. EVOLUTION OF THE DISCHARGE

In the lowest-order approximation, with respect to j_e and $E-E_{\rm br}$ where $E_{\rm br}$ is the breakdown electric field, the system of equations (23) and (24) has the following form:

$$\frac{\partial j_e(L,t)}{\partial t}=j_e(L,t)\kappa(E_{\rm br})(E-E_{\rm br}), \tag{27}$$

$$\frac{\partial E}{\partial t}=-\chi(E_{\rm br})j_e(L,t), \tag{28}$$

where

$$\kappa(E_{\rm br})=\frac{\partial}{\partial E}\frac{1}{\tau(E)}\bigg|_{E=E_{\rm br}}=\frac{v_i}{L}\frac{\ln(1+1/\gamma)[(\partial\ln\gamma/\partial E)+(\gamma+1)L(\partial\alpha/\partial E)]|_{E=E_{\rm br}}}{(\gamma+1)\ln(1+1/\gamma)-1} \tag{29}$$

and

$$\chi(E_{\rm br})=\frac{4\pi d}{\epsilon L+2d}[1+1/\gamma+e^{\alpha(E_{\rm br})L}]$$

$$\approx\frac{8\pi d}{\epsilon L+2d}(1+1/\gamma). \tag{30}$$

We can find the differential equation for the $I-V$ curve if we divide Eq. (27) by Eq. (28). We have

$$\frac{\partial j_e(L,t)}{\partial E}=-\frac{\kappa(E_{\rm br})}{\chi(E_{\rm br})}[E(t)-E_{\rm br}]. \tag{31}$$

This equation is easy to integrate and we obtain

$$j_e(L,t)+\frac{1}{2}\frac{\kappa(E_{\rm br})}{\chi(E_{\rm br})}[E(t)-E_{\rm br}]^2=\text{const}=j_e(L,t)_{\max}. \tag{32}$$

The value of this constant is determined by the initial conditions. If we initiate the discharge with low electron and ion densities, then $j_e(L,0)$ is much less than $j_e(L,t)_{\max}$ and $j_e(L,t)_{\max}$ is given by

$$j_e(L,t)_{\max}=\frac{1}{2}\frac{\kappa(E_{\rm br})}{\chi(E_{\rm br})}(E_0-E_{\rm br})^2. \tag{33}$$

Here E_0 is the electric field in the gap at time $t=0$: $E_0=U_{\rm gap}(t=0)/L$. Figure 2 shows a plot of Eq. (32) which is essentially the dynamic $j-U$ curve for the discharge. Each point $[U(t),j(t)]$ of this curve is a solution of Eqs. (27) and (28) and during the discharge this point moves along this curve in the direction shown by arrows, so that voltage in the gap can only decrease. As seen from this plot and from Eq. (32) the value of the voltage is symmetrical about the breakdown voltage. Thus

$$\Delta E(t=\infty)=E(t=\infty)-E(0)=-2(E_0-E_{\rm br})$$

or

$$\Delta U_{\rm gap}(\infty)=-2[U_{\rm gap}(0)-U_{\rm br}]. \tag{34}$$

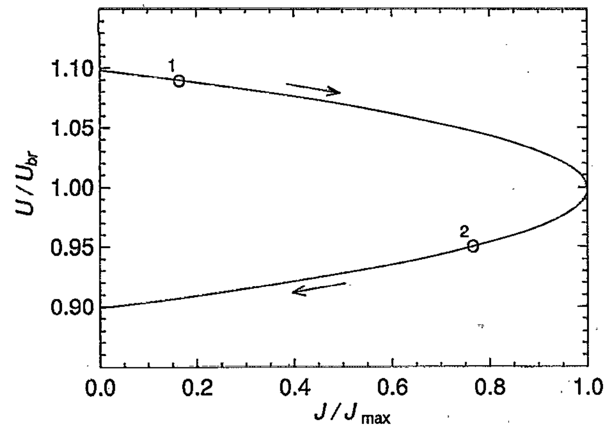

FIG. 2. $j-U$ curve [Eq. (32)] of a pdp cell near the breakdown threshold. Voltage normalized to a breakdown voltage vs current density normalized to the maximum current density. Each point (j,U) of this curve moves along the curve during the discharge, in the direction shown by arrows (the point where current density reaches its maximum is related to a breakdown voltage). If initial voltage is below the breakdown voltage (point 2 on the curve), then the maximum current is the one at this initial point.

Later, when investigate the stability of the discharge, we will exploit this fact.

Now that we have derived an expression for the $j-U$ curve, we solve Eqs. (27) and (28) for $j_e(L,t)$ and $E(t)$ explicitly. Substitution of Eq. (32) into (27) gives

$$\frac{\partial j_e(L,t)}{\partial t}=\pm\Omega j_e(L,t)\sqrt{1-j_e(L,t)/j_e(L,t)_{\max}}, \tag{35}$$

where

$$\Omega=\sqrt{2\chi(E_{\rm br})\kappa(E_{\rm br})j_e(L,t)_{\max}}=\kappa(E_{\rm br})(E_0-E_{\rm br}). \tag{36}$$

The plus and minus signs in Eq. (35) are related to the growing and decaying parts of the current, respectively. Integration of the Eq. (35) yields

$$t=\frac{1}{\Omega}\int_{j_0/j_{\max}}^{j_{eL}/j_{\max}}\frac{dx}{x\sqrt{1-x}} =t_m+\frac{1}{\Omega}\ln\left(\frac{1-\sqrt{1-j_{eL}/j_{\max}}}{1+\sqrt{1-j_{eL}/j_{\max}}}\right) \tag{37}$$

for $t\leqslant t(j_{\max},j_0)$, where

$$t_m\equiv t(j_{\max},j_0)=\frac{1}{\Omega}\int_{j_0/j_{\max}}^{1}\frac{dx}{x\sqrt{1-x}} =\frac{1}{\Omega}\ln\left(\frac{1+\sqrt{1-j_0/j_{\max}}}{1-\sqrt{1-j_0/j_{\max}}}\right) \tag{38}$$

and

$$t=t_m+\frac{1}{\Omega}\int_{j_{eL}/j_{\max}}^{1}\frac{dx}{x\sqrt{1-x}} =t_m+\frac{1}{\Omega}\ln\left(\frac{1+\sqrt{1-j_{eL}/j_{\max}}}{1-\sqrt{1-j_{eL}/j_{\max}}}\right) \tag{39}$$

for $t\geqslant t(j_{\max},j_0)$. Here, for brevity, we suppressed the time dependence in the current and introduced the notation $j_{eL}\equiv j_e(L,t)$, $j_0\equiv j_e(L,0)$, and $j_{\max}\equiv\max[j_e(L,t)]$.

Usually $j_{\max}$ is two to three orders of magnitude larger than j_0, in which case Eq. (38) becomes

$$t_m=\frac{1}{\Omega}\ln\left(\frac{4j_{\max}}{j_0}\right)\equiv\frac{\Lambda}{\Omega}. \tag{40}$$

Typically Λ is about 5–10, and does not change much because it is a logarithm of a big number. It is convenient to rewrite Eqs. (37) and (39) in the following form:

$$\Omega|t-t_m|=\ln\left(\frac{1+\sqrt{1-j_{eL}/j_{\max}}}{1-\sqrt{1-j_{eL}/j_{\max}}}\right). \tag{41}$$

Hence

$$\frac{j_{eL}}{j_{\max}}=1-\tanh^2\left[\frac{\Omega(t-t_m)}{2}\right]\equiv\operatorname{sech}^2\left[\frac{\Omega(t-t_m)}{2}\right]. \tag{42}$$

Equation (42) together with Eqs. (33), (36), and (38) [or (40)] give the solution for $j_e(L,t)$ which we sought.

We now solve for $E(t)$. Substituting Eq. (42) into Eq. (32) and using Eq. (33) we obtain the electric field:

$$E(t)-E_{\rm br}=\sqrt{\frac{2\chi(E_{\rm br})j_{\max}}{\kappa(E_{\rm br})}}\tanh\left[\frac{\Omega(t-t_m)}{2}\right] =(E_0-E_{\rm br})\frac{\tanh[\Omega(t_m-t)/2]}{\tanh(\Omega t_m/2)}. \tag{43}$$

The plots of the electric field in the gap $[E(t)]$ and the total discharge current density (which is independent of z)

$$J(t)=(1+1/\gamma)j_e(L,t)=J_{\max}\operatorname{sech}^2\left[\frac{\Omega(t-t_m)}{2}\right],$$

are shown in Fig. 3. Here $J_{\max}=(1+1/\gamma)j_{\max}$ is the maximum value of the total current density.

It should be noted that Eqs. (42) and (43) remain valid when $E_0<E_{\rm br}$; however, Ω and t_m become negative and $j_{\max}$ should not be interpreted as a maximum current, but only as a constant defined by Eq. (32). This is because if initial voltage is lower than the breakdown one, the current cannot increase (the direction of the motion along the $J-U$ curve is determined). Thus in this case initial current is the maximum one, and the value $j_{\max}$ determined by formula (32) should be interpreted only as a parameter of the $J-U$ curve (compare

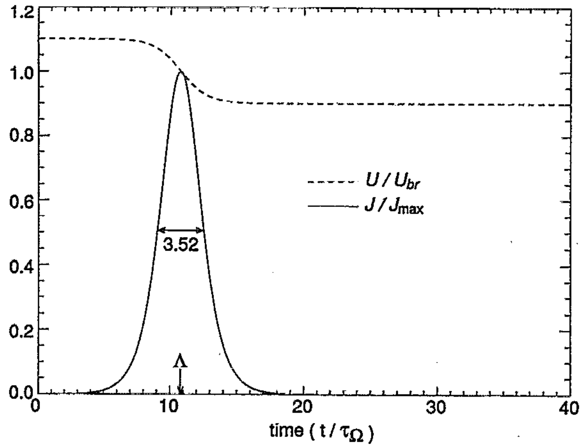

FIG. 3. Current density (solid line) and gap voltage (dashed line), normalized to the maximum discharge current [Eq. (33)] and breakdown voltage, respectively, vs time, normalized to the characteristic time τ_Ω [Eq. (44)], during the discharge in a pdp cell.

points 1 and 2 in Fig. 2). Solutions (42) and (43) for the current and electric field show that a characteristic time τ for this problem can be defined as

$$\tau=\tau_\Omega\equiv\frac{1}{|\Omega|}=\frac{1}{\kappa(E_{\rm br})|E_0-E_{\rm br}|}. \tag{44}$$

The half-width of the current pulse is determined by the conditions $j_{eL}/j_{\max}=0.5$, which yields

$$\tau_{1/2}=3.52\tau_\Omega\equiv\frac{3.52}{|\Omega|}. \tag{45}$$

It is insightful to note the analogy between Eqs. (27) and (28) and Hamilton's equations for one-dimensional motion. Let us choose variables

$$q=\ln j_e(L,t), \quad p=E-E_{\rm br}. \tag{46}$$

In terms of these variables, Eqs. (27) and (28) have the form of Hamilton's equations for the motion of a particle with a mass equal to $\kappa^{-1}(E_{\rm br})$ in the potential $V(q)=\chi(E_{\rm br})e^q$. In this case,

$$\dot q=\kappa(E_{\rm br})p, \quad \dot p=-\chi(E_{\rm br})e^q. \tag{47}$$

The energy equation for these equations is

$$\mathfrak{E}=\frac{1}{2}\frac{\dot q^2}{\kappa(E_{\rm br})}+\chi(E_{\rm br})e^q=\text{const}, \tag{48}$$

which is essentially our dynamic $I-V$ curve [see Eq. (32)]. Solving Eq. (47) for $\dot w$ and integrating over time, we obtain the time dependence of the current and electric field in the gap:

$$t=\int_{q(0)}^{q(t)}\frac{|dq|}{\sqrt{2\kappa(E_{\rm br})}\sqrt{\mathfrak{E}-V(q)}}=\frac{1}{\sqrt{2\kappa(E_{\rm br})\chi(E_{\rm br})}}\int_{q(0)}^{q(t)}\frac{|dq|}{\sqrt{[\mathfrak{E}/\chi(E_{\rm br})]-e^q}}. \tag{49}$$

Here, as in Eqs. (37)–(39), the integration must be performed along the "trajectory." Note that the trajectory has a "turning point" if $p(t=0)>0$ or $E(t=0)>E_{\rm br}$ and does not have such a point in the opposite case, when particle velocity ($\dot q$) and acceleration ($\dot p$) are collinear at $t=0$.

As a particular example of the theory developed above, we consider a discharge in a plasma display picture element cell. We estimate the characteristic time (45) for a typical set of parameters of the discharge, the time delay t_m [Eq. (40)] between applying the voltage and the time when the current reaches its maximum value and the half-width and the maximum value of the current. For this example, we choose $L\sim10^{-2}$ cm, $d\sim0.1L$, $\epsilon\sim10$, and $\gamma\sim0.25$. From Eq. (1) we immediately find $\alpha\approx160\ \text{cm}^{-1}$, which determines the breakdown voltage (electric field) for any specific gas. For He at a pressure $p\sim400$ Torr, using the formula[7,8]

$$\alpha=Cp\exp(-D\sqrt{p/E}) \tag{50}$$

for α in noble gases with $C=4.4\ \text{Torr}^{-1}\text{cm}^{-1}$ and $D=14[\text{V/(cm Torr)}]^{1/2}$, we obtain

$$E_{\rm br}=\frac{D^2p}{\ln^2[CpL/\ln(1+1/\gamma)]}=1.37\times10^4\ \text{V/cm},$$

or $U_{\rm br}\approx137$ V. When differentiating γ [as required by Eq. (29)] we can use the fact that, for moderate electric fields, the second Townsend coefficient depends on the electric field as[9–11] $\gamma\approx\gamma_{i\nu}/(1+\tilde E/E)$. Here $\gamma_{i\nu}$ is the vacuum value of the secondary electron emission coefficient and $\tilde E$ is about the ratio of the initial energy of an emitted electron at the surface and electron mean free path, $\tilde E\sim W_0/(2e\lambda_e)$. In our example the gas density is $N=1.4\times10^{19}\ \text{cm}^{-3}$, the momentum transfer cross section is $\sigma_{\rm mt}\approx5\times10^{-16}\ \text{cm}^2$, $\lambda_e=(N\sigma_{\rm mt})^{-1}\sim1.4\times10^{-4}$ cm, and $E/\tilde E$ is about $4/W_0$. We have not found any reliable data about the energy spectrum of the emitted electrons. At the metal surface W_0 is about 10 eV, but at the dielectric surface it may be significantly lower. Assuming $E\approx\tilde E$, we have $(1/\gamma)(\partial\gamma/\partial E)=1/2E$. Substituting the former into Eq. (29) and using Eq. (50) for α yields

$$\Omega=\frac{v_i}{L}\frac{E_0-E_{\rm br}}{E_{\rm br}}\frac{\ln(1+1/\gamma)\left[1/2+(\gamma+1)\ln(1+1/\gamma)\,\frac{D}{2}\sqrt{p/E_{\rm br}}\right]}{(\gamma+1)\ln(1+1/\gamma)-1}=\frac{\mu_i(E_0-E_{\rm br})}{2L}\frac{\ln(1+1/\gamma)\{1+(\gamma+1)\ln(1+1/\gamma)\ln[CpL/\ln(1+1/\gamma)]\}}{(\gamma+1)\ln(1+1/\gamma)-1}. \tag{51}$$

For the chosen parameters, $\Omega=4.6[\mu_i(E_0-E_{br})/L]$. Choosing $(E_0-E_{br})/E_{br}=0.1$ and $\Lambda=9.2$, we obtain $\Omega\approx 0.95\times 10^7$ s^{-1} and $t_m\approx 1$ μs [see Eq. (39)]. The half-width of the current pulse is $\tau_{1/2}=375$ ns. One can find a convenient expression for the total current at maximum using expressions (33) and (36):

$$J_{max}=(1+1/\gamma)j_{max}=(1+1/\gamma)\,\frac{\Omega}{2\chi(E_{br})}\,(E_0-E_{br}). \tag{52}$$

Using the parameters we have chosen for our example, we find $J_{max}=28$ mA/cm^2.

V. THE STABILITY OF OPERATION NEAR THE THRESHOLD

An important concern for any periodic system is the stability within the operational regime and we now consider the stability of the discharge parameters during a sequential firing. When the alternating voltage is applied to cell electrodes, the cell periodically fires. We assume that the half-period (T) of the external voltage wave form is larger than the discharge pulse time ($2t_m$). For our example of a pdp element, T is about $(3-10)\times 10^{-6}$ s, while $2t_m$, as we know from the above example, is only about 10^{-6} s. In this case the shape and time dependence of each pulse are described by Eqs. (37), (42), and (43).

First, let us clarify what we mean by stability. Suppose we have a square-wave voltage with an amplitude U_0 applied to a cell (see Fig. 1). For certainty we choose that one of the electrodes (i.e., right) is grounded. According to our solution [Eq. (13)] for the electric field in the gap, the voltage across the gap $U_{gap}=EL$ at any time can be represented as a sum of two terms. One of these terms is independent of the charge on the plates and caused by external voltage only:

$$U_0'=U_0\,\frac{L}{L+2d/\epsilon}\approx U_0. \tag{53}$$

The other one originates from the charge accumulated on the dielectric surfaces:

$$U_\sigma=\frac{4\pi d}{\epsilon+2d/L}\,(\sigma_a-\sigma_c)=\frac{8\pi d}{\epsilon+2d/L}\,\sigma_a\equiv\frac{8\pi d}{\epsilon+2d/L}\,\sigma. \tag{54}$$

Let us assume now that at the time $t=t_1$ the external voltage changes its polarity so that $U(t_1)=U_0$ is positive and there is a positive free charge σ_1 on the left surface (and correspondingly $-\sigma_1$ on the right). If the total voltage across the gap,

$$U_{gap}(t_1)=U_0'+U_{\sigma 1}, \tag{55}$$

exceeds the breakdown voltage U_{br}, then the discharge current grows, reaches its maximum at $t=t_1+t_m$, and then quickly decreases (compared to T). At the time $t\approx t_1+2t_m$ the discharge is practically extinguished. At this moment the gap voltage is decreased by the value ΔU_{gap} due to a charge deposition on the dielectric surfaces during this pulse, and $U_{gap}(t_1)-\Delta U_{gap}<U_{br}$. The value of ΔU_{gap} depends on the total voltage across the gap at the time t_1 and we introduce the transfer function f (Refs. 12 and 13) equal to this change ΔU_{gap} as a function of the total voltage across the gap at the time when the applied voltage just changes its polarity. Thus

$$\Delta U_{gap}\equiv\Delta U_{gap}(U_{gap},U_{br})\equiv f(U_{gap},U_{br}). \tag{56}$$

The new voltage due to a charge $U_{\sigma 2}$ is equal to

$$U_{\sigma 2}=U_{\sigma 1}-f[U_{gap}(t_1),U_{br}]. \tag{57}$$

When $t=t_2=t_1+T$, the external voltage changes its polarity and the total voltage applied to the gap becomes equal to

$$U_{gap}(t_2)=-U_0'+U_{\sigma 2}. \tag{58}$$

Since we seek periodic operation of a cell we must require the following conditions:

$$U_{gap}(t+T)=-U_{gap}(t),\quad U_{gap}(t+2T)=U_{gap}(t). \tag{59}$$

Applying these conditions to Eqs. (55) and (58), we obtain the conditions of the periodicity in the form $U_{\sigma 2}=-U_{\sigma 1}$ or

$$\Delta U_{gap}=2U_\sigma=2(U_{gap}-U_0'). \tag{60}$$

Simultaneous solution of the system of equations (56) and (60) determines the value of U_{gap} (or U_σ) necessary to satisfy the periodic conditions (59) as a function of U_0'. Combining Eqs. (56) and (60), we obtain one equation for determining U_{gap}:

$$f(U_{gap},U_{br})=2U_\sigma=2(U_{gap}-U_0'). \tag{61}$$

Let us assume that at some moment there is an initial difference δU_{gap} between U_{gap} and the solution $U_{gap}(U_0')$ of Eq. (61). For example, fluctuations in the applied voltage can lead to this situation. We say that operational regime is stable if $|\delta U_{gap}|$ decreases with time and unstable if it increases with time. For a pdp element this problem was investigated earlier[12,13] in terms of the transfer function. The result is that a solution $U_{gap}(U_0')$ is stable if

$$0<\left.\frac{\partial f(x)}{\partial x}\right|_{x=U_{gap}(U_0')}<2. \tag{62}$$

Let us now apply this to the results we obtained in the previous sections. We have found that the transfer function is a function of the difference $U_{gap}-U_{br}$ [see Eq. (34)]:

$$f(U_{gap},U_{br})=f(U_{gap}-U_{br})=2(U_{gap}-U_{br}). \tag{63}$$

Comparison of Eq. (56) with the transfer function (63) and Eq. (60) shows that our system of equations (56) and (60) has no solutions for U_{gap} if $U_0'\neq U_{br}$ [Eq. (61) cannot be satisfied], and an infinite number of solutions for U_{gap} if $U_0'=U_{br}$ [Eq. (61) is always satisfied]. This situation is, obviously, unstable with respect to a fluctuation of U_0'. The degeneracy of this case is a direct consequence of the linearity [Eq. (63)] of the transfer function in the vicinity of the breakdown. Any additional nonlinear term in $f(x)$ removes this degeneracy, so we have to take into account such terms. Let us expand the $f(x)$ up to the second order of x near the threshold (here $x=U_{gap}-U_{br}$):

$$f(x)=2x+\tfrac{1}{2}\,\omega x^2, \tag{64}$$

where $\omega=\partial^2 f(x)/\partial x^2|_{x=0}$. Using Eq. (61) we obtain

$$2x+\tfrac{1}{2}\,\omega x^2=2(x+U_{\rm br}-U_0'), \tag{65}$$

which has the solution

$$x\equiv U_{\rm gap}-U_{\rm br}=2\sqrt{\frac{U_{\rm br}-U_0'}{\omega}}. \tag{66}$$

This solution is unstable if $\omega>0$, and stable if $\omega<0$. However, if $\omega<0$ the operation requires a sustain voltage U_0' higher than the breakdown voltage $U_{\rm br}$, and hence, in this case, we do not have an "off" state.

To find the first nonlinear term in $f(x)$ we should return to Eqs. (27) and (28) and save the terms of the order of $\dot{E}$ in the equation for $\partial j/\partial t$ and $(E-E_{\rm br})(\partial\alpha/\partial E)$ in the equation for $\partial E/\partial t$. Doing this yields the following form of the system of equations (27) and (28):

$$\frac{\partial j_e(L,t)}{\partial t}=j_e(L,t)\kappa(E_{\rm br})\xi-j_e(L,t)\,\frac{\partial E}{\partial t}\,\psi, \tag{67}$$

$$\frac{\partial E}{\partial t}=-\chi(E_{\rm br})j_e(L,t)(1+\beta\xi), \tag{68}$$

where $\xi=E-E_{\rm br}$,

$$\beta=\frac{L}{2}\,\frac{\partial\alpha}{\partial E}\bigg|_{E=E_{\rm br}},$$

$$\psi=\left[\frac{1}{\alpha}\,\frac{\partial\alpha}{\partial E}\left(1+\alpha v_i\,\frac{\langle\tau_i^2\rangle}{\langle\tau_i\rangle}\right)-\frac{1}{E}\right]_{E=E_{\rm br}}, \tag{69}$$

and

$$\langle\tau_i^n\rangle=\gamma\int_0^L\tau_i^n(z)\,\alpha e^{\alpha z}\,dz. \tag{70}$$

Here we denoted $\tau_i(z)\equiv z/v_i$. Note that both β and ψ are positive. Dividing Eq. (67) by Eq. (68) we obtain the following equation for $j(\xi)$:

$$\frac{\partial j}{\partial\xi}+\psi j=-\frac{\kappa(E_{\rm br})}{\chi(E_{\rm br})}\,\xi(1-\beta\xi), \tag{71}$$

with the "initial" condition $j(\xi_0)=0$, where $\xi_0\equiv E_0-E_{\rm br}$. Integration of this equation gives

$$j(\xi)=(1+\psi\xi)\,\frac{\kappa(E_{\rm br})}{2\chi(E_{\rm br})}\,(\xi_0-\xi)\bigg(\xi_0+\xi-\frac{2}{3}\,(\beta-\psi)\times(\xi_0^2+\xi^2+\xi_0\xi)\bigg). \tag{72}$$

The second root of the equation $j(\xi)=0$ (the first is $\xi_1=\xi_0$) determines parameter ϵ. Since it must be close to "$-\xi_0$" we represent it as $\xi_2=-\xi_0+\delta\xi$. In the lowest order in $\beta\xi$ and $\psi\xi$ we have

$$\delta\xi=-\tfrac{2}{3}\,(\psi-\beta)\xi_0^2. \tag{73}$$

Thus

$$f(x)=2x+\tfrac{2}{3}\,(\psi-\beta)x^2. \tag{74}$$

Let us now consider the sign of the ω near the threshold:

$$\frac{3}{4}\,\omega=\psi-\beta=\left[\frac{1}{\alpha}\,\frac{\partial\alpha}{\partial E}\left(1+\alpha v_i\,\frac{\langle\tau_i^2\rangle}{\langle\tau_i\rangle}-\frac{\alpha L}{2}\right)-\frac{1}{E}\right]_{E=E_{\rm br}}. \tag{75}$$

With an accuracy of a few percent we can write, for a wide range of αL,

$$\frac{\langle\tau_i^2\rangle}{\langle\tau_i\rangle}=0.7\,\frac{L}{v_i},$$

and using $\alpha L=\ln(1+\gamma^{-1})$ we get

$$\psi-\beta=\left[\frac{1}{\alpha}\,\frac{\partial\alpha}{\partial E}\,(1+0.2\,\ln(1+\gamma^{-1}))-\frac{1}{E}\right]. \tag{76}$$

To be specific, let us again consider a discharge in He. Substituting Eq. (50) into Eq. (76), we obtain that $\omega=0$ for electric field

$$\frac{E_c}{p}=\left[\frac{D}{2}\,(1+0.2\,\ln(1+\gamma^{-1}))\right]^2$$
$$=49(1+0.2\,\ln(1+\gamma^{-1}))^2.$$

For $\gamma\sim0.25-0.3$ this gives $E_c/p\approx85$ V/Torr. If the breakdown electric field is less than this value, then $\omega>0$ and a stationary solution for the amplitude of the voltage across the gap is unstable. In the opposite case ($E>E_c$), it is stable, but it exists only for applied voltages higher than the breakdown one. In the example we considered in the end of Sec. IV, $E_{\rm br}/p$ was only about 35; thus the periodic operation would be unstable in that case.

VI. THE ROLE OF METASTABLES

We now consider how metastable atoms may influence the discharge during a sequential firing. Metastables can influence the dynamics of a discharge because they can serve as a source of ions and electrons through the following processes:

$$A_m+e\rightarrow A^++e+e, \tag{77}$$

$$A_m+\text{wall}\rightarrow A+\gamma_m e, \tag{78}$$

$$A_m+A_m\rightarrow A+A^++e, \tag{79}$$

where γ_m is the secondary emission coefficient for metastables (number of secondary electrons produced on the surface per incident metastable).

The rates of all of these reactions are proportional to either the metastable density N_m or N_m^2; hence we can neglect them if the density of metastables is not very large (the meaning of large will be defined later). This is correct if we consider a single pulse with a low initial density of metastables, or if there is some effective mechanism of their loss other than reactions (77)–(79) which does not lead to production of charged particles. The diffusion of metastables to the anode, or out of the discharge region, or the electron excitation of a metastable with successive photon emission are a few examples of metastable loss without the production of charged particles.

However, *it is easy to imagine conditions when the density of metastables is large enough so one cannot neglect their influence on the discharge. These conditions can be realized during the stable operation of the pdp cell when metastables accumulate in a cell during many pulses. They may reach such a high value that their production and decay during one half-period of the applied voltage become equal. Although the detailed analysis of this situation is beyond the scope of this paper, it still useful to discuss *qualitatively* the possible influence of metastables on the dynamics of the discharge.

To evaluate the effect of metastables we should compare the rates associated with Eqs. (77)–(79) with the ionization rate of electron–atom (in a ground state) collisions during the discharge and with collisions of ions on the cathode surface. Let us denote the cross section for the ionization of the metastable as σ_m^i and the rate of the second process [see Eq. (78)] as k. The electron production rate including reactions (77)–(79) is

$$\dot{n}_e \sim N_0 n_e \langle \sigma_i v \rangle_e + N_m n_e \langle \sigma_m^i v \rangle_e + \frac{1}{2} k N_m^2 + \gamma_m G_1 \frac{N_m}{\tau_D}, \tag{80}$$

where $n_e = n_e(t)$ is the solution for the electron density obtained in Sec. IV (without metastables), σ_i the ionization cross section of an atom in the ground state by electron impact, v the electron velocity, and τ_D a time of losses for metastables due to their diffusion to a wall. The brackets $\langle \ldots \rangle_e$ mean averaging with the electron distribution function, and G_1 is a geometrical factor giving the fraction of the metastable diffusive losses reaching the cathode. The density of metastables N_m can be determined from the following balance equation:

$$\dot{N}_m = N_0 n_e \langle \sigma_m v \rangle_e - N_m n_e [\langle \sigma_m^i v \rangle_e + \langle (\sigma_m^* + \sigma_{sl}) v \rangle_e] - k N_m^2 - \frac{N_m}{\tau_D}, \tag{81}$$

where σ_m^* is a cross section of electron excitation of metastables, and σ_{sl} is a cross section of collisions in which electrons deexcite a metastable, returning it to the ground state (superelastic collisions). Integrating this equation over a half-period (T) of the applied voltage and equating this integral to zero, we find the equation for determining the quasistationary density of metastables:

$$N_0 \langle \sigma_m v \rangle_e n_{em} \tau_{1/2} = (\langle \sigma_m^i v \rangle_e + \langle (\sigma_m^* + \sigma_{sl}) v \rangle_e) N_m n_{em} \tau_{1/2} + T k N_m^2 + \frac{T}{\tau_D} N_m. \tag{82}$$

In the above $\tau_{1/2}$ is a half-width of the current pulse [see Eq. (45)], and n_{em} is the maximum electron density during the pulse.

We now show that we can neglect the terms containing σ_{sl} and σ_m^i compared to the term containing σ_m^* in Eq. (82). Indeed, since the electric field during the pulse is relatively high,

$$eE\lambda_e > W_{\rm ex} \sqrt{\frac{3m}{M}}, \tag{83}$$

the distribution function of electrons is slowly varying at the energies below the excitation threshold ($W_{\rm ex}$), and drops fast at energies above it.[14,15] In the same energy range (below $W_{\rm ex}$) all three cross sections—σ_m^i, σ_m^*, and σ_{sl}—may be treated as constants.[16] Hence the ratio of these terms is about $\langle \sigma_{sl} v \rangle_e : \langle \sigma_m^* v \rangle_e : \langle \sigma_m^i v \rangle_e \sim \sigma_{sl} : \sigma_m^* : \sigma_m^i$. Typically $\sigma_{sl} \ll \sigma_m^i \ll \sigma_m^*$ which justifies our assumption (for example, in He,[17] $\sigma_{sl} \sim 10^{-17}$, $\sigma_m^i \sim 10^{-15}$, and $\sigma_m^* \sim 10^{-14}$ cm^2).

Let us now compare the remaining terms on the right-hand side of Eq. (82) with the source term (left-hand side term) to find the metastable densities at which each of these terms becomes important. One can estimate the excitation rate $\langle \sigma_m v \rangle_e$ for moderate fields as[15]

$$\langle \sigma_m v \rangle_e \sim v_d \frac{eE}{N_0 W_{\rm ex}}. \tag{84}$$

Comparison of the source term $N_0 \langle \sigma_m v \rangle_e n_{em} \tau_{1/2}$ with the excitation term $N_m \langle \sigma_m^* v \rangle_e n_{em} \tau_{1/2}$ shows that the excitation term is important when the metastable density is about or exceeds a value of $N_{m,i}$, defined as follows:

$$\frac{N_{m,i}}{N_0} \sim \frac{\langle \sigma_m v \rangle_e}{\langle \sigma_m^* v \rangle_e} \sim \frac{\sigma_{\rm mt}}{\sigma_m^*} \left(\frac{eE\lambda_e}{W_{\rm ex}} \right)^2 = 4 \frac{\sigma_{\rm mt}}{\sigma_m^*} \left(\frac{v_d}{v_{\rm ex}} \right)^2. \tag{85}$$

Here $\sigma_{\rm mt}$ is the momentum-transfer cross section, and we used $\langle \sigma_m^* v \rangle_e \sim \sigma_m^* \langle v \rangle_e \sim \sigma_m^* v_{\rm ex}/2$, where $v_{\rm ex}$ is electron speed at energy $W_{\rm ex}$. Typically this ratio is about 10^{-3}. In our example for He (used in Sec. IV), $\sigma_m^* \sim 10^{-14}$ cm^2, $\sigma_{\rm mt}/\sigma_m^* \sim 3 \times 10^{-2}$, $\langle \sigma_m v \rangle_e \sim 10^{-9}$ cm^3 s^{-1}, and we see that $\langle \sigma_m v \rangle_e / \langle \sigma_m^* v \rangle_e \sim 10^{-3}$. This means that the electron excitation of the metastables may play a significant role only at very high densities of metastables. The other terms in Eq. (82) become important at much lower metastable densities.

The "annihilation" term (proportional to N_m^2) is important if the metastable density exceeds a value $N_{m,a}$:

$$\frac{N_{m,a}}{N_0} = \sqrt{\frac{n_{em}}{N_0} \frac{\tau_{1/2}}{T} \frac{\langle \sigma_m v \rangle_e}{k}}. \tag{86}$$

One can express the product $n_{em} \tau_{1/2}$ in terms of the voltage drop ΔU during the main pulse as

$$n_{em} \tau_{1/2} = \frac{\bar{j}_{em} \tau_{1/2}}{e v_d} = G_\alpha \frac{j_m \tau_{1/2}}{e v_d} \sim G_\alpha \frac{\epsilon \Delta U}{8 \pi e d v_d}, \tag{87}$$

where $\bar{j}_{em}$ is the electron current density averaged across the gap and G_α is a geometrical factor reflecting the difference between $\bar{j}_{em}$ and j_m: $[G_\alpha \equiv \bar{j}_{em}/j_m = (1 - e^{-\alpha L})/\alpha L]$. We also used the fact that the voltage drop due to the electron current is only a half of the total voltage drop ΔU. Substituting the relationships (84) and (87) into (86), we obtain

$$\frac{N_{m,a}}{N_0} = \sqrt{\frac{U^2 \epsilon G_\alpha}{8 \pi L d N_0^2 T k W_{\rm ex}} \frac{\Delta U}{U}} = 1.5 \times 10^{-11} \sqrt{\frac{\epsilon G_\alpha U_{\rm (Volt)}^2}{L d p_{\rm (Torr)}^2} T_{(\mu s)} k W_{\rm ex} \frac{\Delta U}{U}}. \tag{88}$$

Using the data for our pdp example, He at room temperature,[18] $k \approx 4 \times 10^{-9}$ cm^3 s^{-1}, $G_\alpha \approx [(1+\gamma_i)\ln(1$

$+1/\gamma_i)]\sim 0.5$, and choosing $T=5$ μs, we find $N_{m,a}/N_0 \sim 0.57\times 10^{-5}\sqrt{\Delta U/U}$, which is indeed much less than $N_{m,i}/N_0$.

The diffusion term is important when the density of metastables is approximately

$$\frac{N_{m,D}}{N_0}=\frac{\tau_D}{T}\langle\sigma_m v\rangle_e n_{em}\tau_{1/2}$$

$$\sim G_D G_\alpha \frac{L^2\langle\sigma_m v\rangle_e}{12\lambda_m v_g T}\frac{\Delta U}{8\pi d e v_d}, \qquad (89)$$

where λ_m and v_g are the mean free path and speed of a metastable and G_D is another geometrical factor dependent on the diffusion mode. For the main mode between flat parallel electrodes, $G_D=1$. In the above v_g is the same as for ground-state gas atoms, but the mean free path λ_m is smaller than that of gas atoms since the cross sections for a metastables are very large ($\lambda_m\sim 0.4\lambda_g$). Again, for the parameters we have chosen in our example, we have $N_{m,D}/N_0\sim 10^{-5}G_D\Delta U/U$. This value is less than $N_{m,a}$ near the threshold, so the quasi-stationary density of metastables in our example will be determined by a diffusion. However, the annihilation term can have as big a contribution as a diffusion term in the production of electrons, since the factor G_1 may be significantly lower than unity, especially if we take into account 2D and 3D effects.

The above consideration shows that in any case the metastable density is determined by the terms proportional to T in Eq. (82) and that we can neglect the second term in Eq. (80). Hence we can rewrite Eq. (80) qualitatively as

$$\dot{n}_e\sim\langle\sigma_i v\rangle_e N_0 n_e+G\langle\sigma_m v\rangle_e N_0\frac{n_{em}\tau_{1/2}}{T}, \qquad (90)$$

where the factor G describes possible strong losses of metastables due to diffusion and not to the cathode. The last term in Eq. (90), induced by the metastables, is small compared to the first one when the electron density is high (near the peak), but it is the only term when the electron density is low. This term does not affect the characteristic time (Ω^{-1}), but strongly influences the time delay t_m and the voltage drop during the total half-period of applied voltage. Actually, the voltage drop due to the metastable decay may be comparable to or even larger than the voltage drop during the "fast" part of the discharge and is determined by the ratio $G\langle\sigma_m v\rangle_e/\langle\sigma_i v\rangle_e$. If this ratio is small, then the voltage drop during the afterglow is small. In the opposite case, it may be even larger than during the main current pulse. Obviously, this voltage drop due to a metastable decay or diffusion to the cathode may influence the stability of the operation. However, if the system is unstable, then the instability develops earlier than the metastable density reaches the value at which they become important.

VII. SUMMARY

This paper describes the characteristic features of a high-pressure ac discharge between electrodes coated with a thin layer of dielectric material when the applied voltage only slightly exceeds the breakdown voltage. In this case the current never reaches a high magnitude so one may neglect the distortions of the electric field caused by the ions and the problem can be solved analytically (the discussion of the space-charge effect is given in Appendix C). We found analytical expressions for the spatial distribution and time dependence of the electron and ion currents, and a current–voltage ($j-U$) characteristic curve. The temporal dependence of the current and particle densities is determined exclusively by the ion transit time between electrodes, and the parameter $\Delta[\Delta\equiv\gamma(e^{\alpha L}-1)-1]$. This parameter characterizes the increment of production of secondary electrons in one ion transit time. If Δ is small, then the dynamics of the discharge is determined by all the ions in the gap. They all need to be collected at the cathode in order to produce an excessive number of electrons. If Δ is large, then the cathode region itself can produce enough particles to sustain the discharge and the temporary growth of the discharge is more rapid.

A particular example was used to illustrate the utility of our theory, that of a plasma display element. Analysis of the dependence of the $j-U$ curve was used to investigate the stability during sequential firing of the cell near the threshold. It was shown that for typical discharge parameters and in the absence of metastables the cell is unstable near the threshold.

Metastables do not influence the dynamics of the discharge pulse near its maximum. However, they strongly affect the initial current and afterglow current and thus the time delay t_m between applying the voltage and the maximum of the current [see Eq. (40)] and the voltage drop in the afterglow.

Our results provide a good understanding of the processes in the cell during the discharge and afterglow when the cell is fired in an ac mode. These results can be also applied to an initial part of any discharge and will apply until the current density reaches a relatively high value.

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APPENDIX A

The drift velocity reaches its equilibrium value at the distance $l_v\sim W/(eE)$ and in time $t_v\sim l_v m\nu_{tr}/(eE)$, where W is the average energy of the particle and $\nu_{tr}=N_0\langle\sigma_{mt}v\rangle$ is the collision frequency for the momentum transfer, so the latter conditions can be written as

$$|\nabla E/E|^{-1}\gg W/(eE), \quad |\dot{E}/E|\ll(eE)^2/(Wm\nu_{tr}). \qquad (A1)$$

For ions in their own gas one can estimate W as $T_g+eE/(N_0\sigma_{cx})$, where T_g is a gas temperature and σ_{cx} is a charge-exchange cross section. For the case when $eE/(N_0\sigma_{cx})>T_g$ conditions (A1) have the form

$$|\nabla E/E|^{-1} \gg 1/(N_0\sigma_{cx}), \quad |\dot{E}/E| \ll (eEN_0\sigma_{cx}/M)^{1/2}. \tag{A2}$$

Usually, however, $eE/(N_0\sigma_{cx}) \sim T_g$.

For electrons, the value of W can be much larger than for ions, since they loose only about m/M fraction of their energy during an elastic collision with an atom, and if the electric field is high enough [$eE\lambda_e \gg W_{\rm ex}\sqrt{3m/M}$, where $\lambda_e = 1/(N_0\sigma_{\rm mt})$, $W_{\rm ex}$ is the excitation threshold, and m and M are electron and ion masses, respectively], then they can easily reach the excitation energy $W_{\rm ex}$. However, as soon as they reach $W_{\rm ex}$ they can effectively loose their energy in exciting or ionizing collisions with gas atoms. If the electric field is not too strong, $eE\lambda_e \ll W_{\rm ex}\sqrt{3\sigma_{il}/\sigma_{\rm mt}}$, where σ_{il} is total inelastic cross section at an energy roughly $2W_{\rm ex}$, then the electron distribution function cannot spread far beyond the threshold and the electron average energy is of the order of $W_{\rm ex}$. The above consideration also requires that, between two collisions, the electron gains much less energy that it already has. Combining this requirement with conditions (A1), we obtain for the time and length scales of the changing of the electric field

$$\left|\frac{\nabla E}{E}\right|^{-1} \gg \frac{W_{\rm ex}}{eE} = \frac{W_{\rm ex}}{eU} L \gg \lambda_e, \quad \left|\frac{\dot{E}}{E}\right| \ll \frac{(eE)^2}{m v_{tr} W_{\rm ex}} \ll v_{tr}, \tag{A3}$$

and for the magnitude of the electric field we have

$$W_{\rm ex}N_0\sigma_{\rm mt}\sqrt{3m/M} \ll eE \ll W_{\rm ex}N_0\sqrt{3\sigma_{\rm mt}\sigma_{il}};$$

here we substituted $E = U/L$.

APPENDIX B

The solution of Eq. (17) can be obtained using the method of characteristics. Using the boundary condition $j_i(0,t)=0$, we obtain

$$j_i(z,t) = \int_0^z \alpha(t-\tau_i(z,z'))j_e(z',t-\tau_i(z,z')) \times e^{\eta(t-\tau_i(z,z'))(z-z')}\, dz', \tag{B1}$$

where $\alpha(t) \equiv \alpha[E(t)]$ and $\eta = v_i^{-2}(\partial v_i/\partial t) = \dot{E}/(Ev_i)$, and $\tau_i(z,z')$ is the function, determined in the equation

$$z - z' = \int_{t-\tau_i}^{t} v_i[E(t')]dt'. \tag{B2}$$

Substituting the solutions (16) into Eq. (B1) and using the boundary condition for the electron current $j_e(L,t) = \gamma j_i(L,t)$ we obtain an integral equation for determining $j_e(L,t)$:

$$j_e(L,t) = \gamma \int_0^L \alpha[t-\tau_i(z)]j_e[L,t-\tau_i(z)] \times e^{\alpha[t-\tau_i(z)]z} e^{\eta[(t-\tau_i(z))]z}\, dz, \tag{B3}$$

where $\tau_i(z) \equiv \tau_i(L,z)$. After expanding $j_e(L,t-\tau_i(z))$ in the argument of the integral [Eq. (B3)]

$$j_e(L,t-\tau_i(z)) = j_e(L,t) - \frac{\partial j_e(L,t)}{\partial t}\tau_i(z), \tag{B4}$$

we obtain a differential equation for $j_e(L,t)$:

$$\frac{\partial j_e(L,t)}{\partial t} = j_e(L,t)\, \frac{\gamma \int_0^L \alpha(t-\tau_i(z))e^{\alpha(t-\tau_i(z))z}e^{\eta(t-\tau_i(z))z}\, dz - 1}{\gamma \int_0^L \tau_i(z)\alpha(t-\tau_i(z))e^{\alpha(t-\tau_i(z))z}e^{\eta(t-\tau_i(z))z}\, dz}. \tag{B5}$$

Equation (B5) can be greatly simplified if the ion transit time $\tau_i \sim L/v_i$ is much less than the characteristic time τ: $\tau_i \ll \tau$. In this case we can find from Eq. (B2) $\tau_i(z) = (L-z)/v_i$ and neglect all the terms containing $\dot{E}$, like $\dot{\alpha}\tau_i L$ and η. After this, the integration in Eq. (B5) becomes very simple and we obtain Eq. (23).

APPENDIX C

Space-charge effect

Here we show that the space charge has a small effect on the dynamics of the discharge in a pdp cell near the threshold. The main effects caused by the space charge at the beginning of the discharge, when the current is still small, are a changing of the value of the integral $\int \alpha(E)dz$, due to distortion of the electric field in the gap and change of γ, which depends on the value of the electric field in the vicinity of a cathode. Expanding $\alpha(E)$ up to the second order with respect to the difference $E(z,t) - E_{\rm br}$, which we consider small compared to the $E_{\rm br}$,

$$\alpha(E) = \alpha(E_{\rm br}) + (E-E_{\rm br})\alpha' + \tfrac{1}{2}(E-E_{\rm br})^2\alpha'', \tag{C1}$$

and introducing the space-charge distortion of the electric field $\delta E(z,t)$,

$$E(z,t) = \bar{E}(t) + \delta E(z,t), \tag{C2}$$

such that

$$U(t) = \bar{E}(t)L, \quad \int_0^L \delta E(z,t)dz = 0, \tag{C3}$$

where $U \equiv U(t) = \int_0^L E(z,t)dz$ is the voltage across the gap, we obtain

$$\int_0^L \alpha(E)dz = \alpha(E_{\rm br})L + \left[\alpha' + \frac{1}{2}\alpha''(\bar{E}-E_{\rm br})\right](U-U_{\rm br}) + \frac{1}{2}\alpha''\int_0^L (\delta E)^2\, dz. \tag{C4}$$

Similarly, for γ we have

$$\gamma = \gamma(E_{\rm br}) + \gamma'[(\bar{E}-E_{\rm br}) + \delta E(L)]. \tag{C5}$$

In order to simplify notations we dropped the arguments of $\delta E(z,t)$ and $U(t)$ in these expressions and used a "prime" to denote the E derivative $\partial/\partial E$ at the $E = E_{\rm br}$. The terms proportional to $U - U_{\rm br}$ in Eq. (C4) and to $\bar{E} - E_{\rm br}$ in Eq. (C5) reflect the influence of the charges accumulated on the plates. They always decrease during a discharge as $\bar{E}$ decreases with time according to Eq. (28). The terms containing δE reflect the influence of the space charges, since they exist only in nonuniform fields and they may increase during a discharge if the nonuniformity of the electric field increases.

In our consideration in Secs. III and IV we neglected terms containing δE and term $1/2\alpha''(\bar{E}-E_{br})^2L$ in the expansions (C4) and (C5). To establish the validity of that approximation, we should include all of these terms in the Eq. (27). First of all, note that the term $1/2\alpha''(\bar{E}-E_{br})^2L$ is practically always small compared to $\alpha'(U-U_{br})$. Indeed, one can neglect it if

$$\left|\frac{1}{2}\,\alpha''(\bar{E}-E_{br})\right| \ll \alpha' \tag{C6}$$

or

$$\left|\frac{1}{2}\,(\bar{E}-E_{br})(\ln\,\alpha')'\right| \ll 1. \tag{C7}$$

For the dependence $\alpha(E)$ of the kind $\alpha(E)\propto\exp[-D(p/E)^{1/n}]$, this gives

$$\left|\frac{\bar{E}-E_{br}}{2E_{br}}\left[\frac{D}{n}\left(\frac{p}{E_{br}}\right)^{1/n}-\left(1+\frac{1}{n}\right)\right]\right| \ll 1. \tag{C8}$$

Thus for any electric field more than

$$\frac{E_{br}}{p} > \left[\frac{D}{n(n+1)}\right]^2, \tag{C9}$$

condition (C6) is equivalent to $|\bar{E}-E_{br}|/E_{br}\ll 1$, which we considered as always being satisfied. For He, $D=14$, $n=2$, and condition (C9) results in $E_{br}/p>5.5$ V/(cm Torr).

Now let us consider the effect of the field distortion. One can find the value of $\delta E(z,t)$ by solving Eq. (5) with condition (C3). Neglecting the electron density compared to the ion density and using condition (2) and solution (12), we can rewrite Eq. (5) in the following form:

$$\frac{\partial}{\partial z}\,\delta E(z,t) \approx \frac{4\pi J(t)}{v_i}\,[1-\exp\,\alpha(z-L)]$$
$$\approx \frac{4\pi(1+\gamma)j_e(L,t)}{\gamma v_i}\,[1-\exp\,\alpha(z-L)], \tag{C10}$$

which has a simple solution:

$$\delta E(z,t) = \frac{4\pi(1+\gamma)j_e(L,t)}{\gamma v_i}\,F(z), \tag{C11}$$

with

$$F(z) = z-\frac{L}{2}+\frac{e^{-\alpha L}}{\alpha}\left[\frac{1}{\alpha L}\,(e^{\alpha L}-1)-e^{\alpha z}\right]. \tag{C12}$$

Substituting Eqs. (C11) and (C12) into integral (C4), we obtain

$$\int_0^L(\delta E)^2\,dz = \frac{4}{3}\left[\frac{\pi(1+\gamma)j_e(L,t)}{\gamma v_i}\right]^2 L^3F_1(\alpha L), \tag{C13}$$

where the function $F_1(\alpha L)<1$ is a dimensionless integral $F_1(\alpha L)=(12/L^3)\int_0^L F^2(z)dz$. With the terms containing $\delta E(z,t)$, Eq. (31) obtains the form

$$\frac{\partial y}{\partial x} = -2x-au_0y+bu_0^3y^3, \tag{C14}$$

where $y=j_e(L,t)/j_{max}=j_e(L,t)[2\chi(E_{br})/\kappa(E_{br})](E_0-E_{br})^{-2}$ is a current density, normalized to its maximum value [Eq. (33)] in the absence of a space charge, $x=(E-E_{br})/(E_0-E_{br})$ is the normalized electric field, $u_0=(E_0-E_{br})/E_{br}$, and coefficients a and b are the following:

$$a=\frac{\ln(1+1/\gamma)}{(1+\gamma)\ln(1+1/\gamma)-1}\,\frac{\epsilon L}{2d}\,\frac{F(L)}{L}\,\frac{\gamma'}{\gamma}\,E_{br},$$

$$b\approx\frac{8}{3}\left(\frac{\pi L(1+\gamma)j_{max}E_{br}}{\gamma v_i(E_0-E_{br})^2}\right)^2\frac{\alpha''(1+\gamma)LE_{br}F_1(\alpha L)}{\alpha' L(1+\gamma)+\gamma'/\gamma}$$
$$=\frac{2}{3}\left(\frac{\epsilon L}{8d}\right)^2\left(\frac{\kappa L}{v_i}\,E_{br}\right)^2\frac{\alpha''E_{br}}{\alpha'}\,\frac{\alpha' L(1+\gamma)F_1(\alpha L)}{\alpha' L(1+\gamma)+\gamma'/\gamma}.$$

Analysis of Eq. (C14) together with Eq. (28) shows that one can neglect space-charge effects if both coefficients au_0 and bu_0^3 are small compared to unity. For the example we considered in the end of the Sec. IV, it gives $(E_0-E_{br})/E_{br}<0.1$. In the case when any of the coefficients au_0 or bu_0^3 is of the order of unity or larger, one should take into account the space-charge effects. With this statement we close our discussion of the space-charge effects in this paper. We are planning to discuss this topic in much more detail in the following paper devoted to a discharge far from the threshold.

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